ASYMPTOTIC REALIZATION OF THE METHOD OF INTEGRAL RELATIONS FOR TURBULENT JETS TO COMPUTE WALL-JETS CURTAIN AND THEIR CONTROL

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The protection of the surface of bodies from high-temperature fluid flows is a real problem in modern technology. The aerodynamic protection is effected with the help of gas curtain. One of the principal means of organizing such a curtain is to blow cold fluid through slots at the initial section of the surface being protected [1]. The computational problem consists in the determination of the surface-temperature distribution in the region of the gas curtain. The existing methods to compute wall-jet curtains are in three principal directions depending on the physical model being used [2]: free turbulent jet model; two-layer scheme with laws on semibounded turbulent jets; flow in a boundary layer with wall-jet curtains determined by wall turbulence. The simplest method appeared to be based on the use of integral relations for the boundary layer and asymptotic conditions when equalization of temperature inside the boundary layer [3] takes place at $x \to \infty$ due to turbulent mixing. Under these conditions, there is a limiting relation between the momentum and energy thicknesses which cannot fully reflect the influence of initial conditions and the previous history of the flow. Expressions for the effect of cooling are developed using interpolations of the type $\eta = (1 + \eta^a_{x\to\infty})^b$, where a and b are constants, $\eta_{x\to\infty}$ is the value of the efficiency obtained on the basis of the laws of boundary-layer growth away from the location of blowing. The model for the wall jet takes into consideration the behavior of mixing processes near the lip of the nozzle [4]. Studies on turbulent boundary layer in wall jets have been carried out in many theoretical and experimental works [5-7]. The majority of theoretical studies is based on the simultaneous solution of the equations for turbulent jets and boundary layer growing on a flat plate, the difference being in the manner of specifying velocity profiles and skin-friction. In studying flows near curvilinear surfaces, disagreement has been noticed between experimental data and theoretical results computed from Karman integral relations which do not take into account the surface curvature in an explicit form. In [8, 9], it has been observed that the effect of surface curvature on the semibounded jet and the efficiency of wall-jet curtain mainly depend on active or conservative role of centrifugal body forces. In the present work, a method is developed to compute the flow and heat transfer on an adiabatic curvilinear surface, based on the asymptotic continuation of the method of integral relations for turbulent wall jets away from the location of blowing. The above-mentioned limiting cases correspond to conditions for the reorganization of nonmonotonous velocity profile which is present near the lip of the nozzle, in the flow that is characteristic of a developed boundary layer [7]. Such an approach makes it possible to develop a computational method for the wall-jet curtain with initial conditions for outflow while computational expressions for the efficiency of film cooling are developed on the basis of interpolation formulas.

1. Problem Formulation. Adiabatic Curvilinear Surface with Gas Curtain. Consider a quasiisothermal turbulent wall jet. It is assumed that the physical characteristics of the fluid are constant in the given temperature range. The basic flow has a velocity u_0 and temperature T_0. The same fluid is blown through a slot of height s with a mean velocity u_S and temperature T_S at the exit section. In the initial segment $0 < x < x_S$ the curvilinear surface is in contact only with the fluid curtain and, hence, has a temperature $T_W = T_S$ (Fig. 1) at all points. The thermal boundary layer begins from the section $x = x_S$ as a result of mixing of the fluid curtain with the basic flow. Equations of quasisteady turbulent, incompressible boundary layer on the curvilinear surface with a constant curvature (neglecting normal turbulent stress components) take the form [10]

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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{kv}{1+ky}=0; \tag{1.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{kuv}{1+ky}=-\frac{1}{1+ky}\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\rho}\frac{\partial \tau}{\partial y}+\frac{2k\tau}{(1+ky)\rho}; \tag{1.2}$$

$$\frac{ku^2}{1+ky}=\frac{1}{\rho}\frac{\partial p}{\partial y},\ \tau=-\rho\langle u'v'\rangle+\mu(1+ky)\frac{\partial}{\partial y}\left(\frac{u}{1+ky}\right); \tag{1.3}$$

$$u\frac{\partial \vartheta}{\partial x}+(1+ky)v\frac{\partial \vartheta}{\partial y}=\frac{1}{\rho}\left[(1+ky)\frac{\partial q}{\partial y}+2kq\right]. \tag{1.4}$$

Here u and v are velocity comonents in the x and y directions, respectively; the coordinate x is measured from the nozzle section and is directed along the tangent to the surface; y, normal to it; p, pressure; $\vartheta = T - T_0$, excess temperature; T, temperature in the boundary layer; T_w and T_0, temperatures of the wall and the fluid away from the curvilinear surface; τ and q, expressions for the total shear stress and heat flux; $k = 1/R$, surface curvature; R, radius of curvature; ρ, density. The only component of the vorticity in two-dimensional flow is its projection along z, perpendicular to the plane of the flow $\zeta_z = -\partial u/\partial y - ku/(1 + ky)$. The boundary conditions are zero vorticity in the main flow, and also the no-slip and adiabatic conditions at the wall

$$u=v=0,\ T=T_{ad,w}\ \text{for}\ \ y=0, \tag{1.5}$$
$$u_0(1+ky)=u_{0,w},\ T=T_0\ \text{for}\ \ y\to\infty,$$

where $u_0(x, y)$ can be considered the velocity distribution in the potential flow past the curvilinear surface; $u_{0,w}(x)$ is the value of velocity u_0 at the wall (see Fig. 1); $T_{ad.w}$ is the wall temperature under adiabatic conditions.

The jet flow field can be split into the primary segment with two characteristic regions based on the jet width δ, δ_m within which velocity and temperature profiles are linear and an initial segment with a core of constant velocities and temperatures [6] (see Fig. 1). The "docking" method is based on approximating the finite thickness boundary layer and momentum and energy integral relations written for the wall layer with velocity u_m at its outer edge, and the jet region of the curtain [10]. The following system of integral relations are obtained by integrating the equations of motion and heat transfer (1.2)-(1.4) from the surface (y = 0) to the point with the ordinate $y = \delta_m$, δ using the equation of continuity (1.1)

$$\frac{d}{dx}\left[\int\limits_{\delta_m}^{\delta}(u^2h^2-u_{0,w}^2)\,udy\right]=-\frac{2}{\rho}\int\limits_{\delta_m}^{\delta}\tau h^2\frac{\partial}{\partial y}(hu)\,dy-\frac{2}{R}\int\limits_{\delta_m}^{\delta}u\frac{\partial}{\partial x}\left(\int\limits_{\delta}^{y}u^2dy\right)dy-(u_m^2-u_{0,w}^2)\frac{d}{dx}\int\limits_{0}^{\delta_m}udy; \tag{1.6}$$

$$(1+\delta_m/R)\frac{d}{dx}\int\limits_{\delta_m}^{\delta}u^2dy-u_{0,w}\frac{d}{dx}\int\limits_{0}^{\delta}udy+u_m(1+\delta_m/R)\frac{d}{dx}\int\limits_{0}^{\delta_m}udy+ \tag{1.7}$$
$$+u_{0,w}^2\frac{d\delta}{dx}\frac{\delta}{R}-u_m^2\frac{d\delta_m}{dx}\frac{\delta_m}{R}-u_{0,w}\frac{du_{0,w}}{dx}\left[(\delta-\delta_m)-\frac{\delta^2-\delta_m^2}{2R}\right]=0;$$

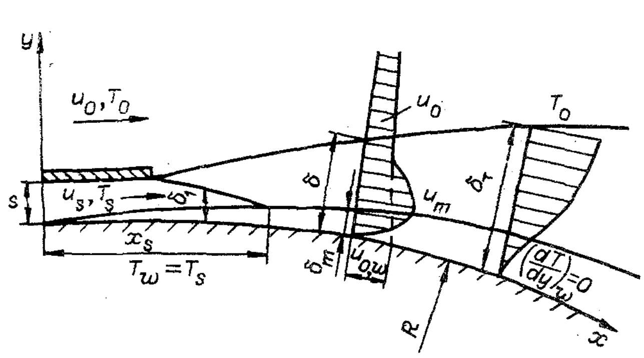

Fig. 1

$$\frac{d}{dx}\int_0^{\delta_m} u^2 dy - u_m \frac{d}{dx}\int_0^{\delta_m} u dy + u_m^2 \frac{d\delta_m}{dx}\frac{\delta_m}{R} - u_{0,w}\frac{\delta_m}{R}\frac{d}{dx}\int_0^{\delta} u dy - \tag{1.8}$$

$$- u_{0,w}\frac{\delta_m}{R}\frac{du_{0,w}}{dx}(\delta - \delta_m) - u_{0,w}\frac{du_{0,w}}{dx}(\delta_m - \delta_m^2/2R) = -\frac{\tau_w}{\rho};$$

$$\frac{d}{dx}\int_0^{\delta_T} \rho u (T - T_0) dy = 0. \tag{1.9}$$

The integral relation (1.6) represents the change in excess energy flux of the wall jet on the curvilinear surface, depending on the strength of turbulent shear stresses, body (centrifugal) forces, and viscous skin friction. It was obtained by multiplying the equation of motion (1.2) and the function uh and the subsequent integration across the thickness of the wall jet, where $h = 1 + y/R$ is the Lamé constant [10]. Expression (1.9) characterizes the conservation of excess enthalpy in the wall jet in the presence of an adiabatic wall. In what follows it will be assumed that the hydrodynamic and thermal boundary layers are approximately equal ($\delta_t = \delta$) [4].

The method of integral relations [5] used to solve the system (1.6)-(1.9) assumes the shear stress relation and the semiempirical approach to determine the turbulent skin friction

$$\tau_w/\rho u_m^2 = A(u_m\delta_m/\nu)^{-m},\ \tau = k\rho(\delta - \delta_m)(u_m - u_0)\frac{\partial u}{\partial y} \tag{1.10}$$

($A = 0.00833$, $m = 2/13$, $k = 0.011$) and velocity profiles in the wall and jet regions expressed in the form

$$u/u_m = \begin{cases} P(\varphi),\ \varphi = y/\delta_m \text{ for } 0 \leqslant y \leqslant \delta_m, \\ Q(\zeta),\ \zeta = (y - \delta_m)/(\delta - \delta_m) \text{ for } \delta_m \leqslant y \leqslant \delta. \end{cases} \tag{1.11}$$

The surface curvature and the velocity of the main flow weakly affect the relative jet velocity profile in the wall as well as in the jet regions [11], and hence the "one-twelfth" law $P(\ ) = (y/\delta_m)^{1/12}$ and the Schlichting profile $Q(\zeta) = (1 - \zeta^{3/2})^2$ [6] are used in the present work. The "depth" of cooling of the adiabatic wall by the gas curtain is determined by the cooling efficiency $\eta = (T_0 - T_{ad.w})/(T_0 - T_s)$. Using the equation for the conservation of excess enthalpy (1.9), the expression for velocity profiles (1.11), and the condition for linear variation in temperature inside the boundary layer on the adiabatic surface (true, strictly speaking, away from the region of blowing as $x \to \infty$), the cooling efficiency $\eta_{x\to\infty}$ can be expressed in the form [4]

$$\eta_{x\to\infty} = \left[\frac{12}{13}\bar{u}_m\bar{\delta}_m\left(1 - \frac{13}{25}\frac{\bar{\delta}_m}{\bar{\delta}}\right) + \frac{9}{28}\bar{u}_m\frac{(\bar{\delta} - \bar{\delta}_m)^2}{\bar{\delta}} + \frac{5}{28}\frac{(\bar{\delta} - \bar{\delta}_m)^2}{m\bar{\delta}}\right]^{-1}, \tag{1.12}$$

where $\bar{u}_m = u_m/u_s$; $\bar{\delta}_m = \delta_m/s$; $\bar{\delta} = \delta/s$; $m = u_s/u_0$. Away from the location of blowing the maximum excess velocity of the jet $\varepsilon = |u_m - u_0|$ tends to zero which corresponds to the transformation of the system (1.6)-(1.8) to the system of integral relations that is true, in general, only when $x \to \infty$. Such an approach at high Reynolds numbers $Re_s = u_s s/\nu$ in the system (1.6)-(1.9) makes it possible to obtain limiting mixing laws for the outer (jet) region of the wall jet. The simultaneous use of two limiting transformations ($Re_s \to \infty$, $x \to \infty$) assumes the satisfaction of the condition $Re_x \to \infty$.

2. Gas Curtain on Adiabatic Flat Plate ($R = \infty$, $u_0 = \text{const}$). Limiting Mixing Laws. Expressions for the determination of the extent of the initial segment x_s and the jet thickness at the end of the initial segment are given in [4]. Integral relations (1.6)-(1.8) in the main segment for the inner and outer regions of the wall jet are transformed to the following form using velocity profiles $P(\eta)$ and $Q(\zeta)$, skin friction relation, and Prandtl's equation (1.10)

$$\frac{d}{dx}[(a_1u_0 + a_2\varepsilon)l] + \varepsilon c_1\frac{d}{dx}(u_m\delta_m) = 0; \tag{2.1}$$

$$\frac{d}{dx}\left[\left(2a_1u_0^2 + 3a_2u_0\varepsilon + a_3\varepsilon^2\right) l\right] + \varepsilon c_1 \frac{d}{dx}(u_m\delta_m)(u_m + u_0) + 2ka_4\varepsilon^3 = 0; \quad (2.2)$$

$$r\frac{dt}{d\xi} + c_3 t\frac{dr}{d\xi} = \frac{13}{15}c_3 r^{-2/13}t^2. \quad (2.3)$$

Here $t = u_m/u_s$; $r = (u_m\delta_m)/(u_s\delta_{m,s})$; $l = (u_m - u_0)(\delta - \delta_m)$;

$$a_i = \int_0^1 Q^i(\zeta)\,d\zeta \ (i = 1, 2, 3); \ c_j = \int_0^1 P^j(\eta)\,d\eta \ (j = 1, 2);$$

$$a_4 = \int_0^1 [Q']^2\,d\zeta; \ c_3 = 1 - c_1/c_2; \ \xi = x/x_s.$$

Expressions for the outer region (2.1) and (2.2) as $\varepsilon \to 0$ and fixed parameter $m = u_s/u_0$ can be simplified by linearization (neglecting products of the order $O(\varepsilon^2)$ and higher) and considering that the thickness of the inner region δ_m is much less than the jet thickness δ:

$$\frac{d}{dx}\left[l\left(a_1u_0 + a_2\varepsilon\right)\right] \to 0, \ \frac{d}{dx}\left[l\left(2a_1u_0^2 + 3a_2u_0\varepsilon\right)\right] \to 0. \quad (2.4)$$

This corresponds to the conservation of excess momentum and kinetic energy for the outer layer of the semibounded jet

$$\frac{d}{dx}\int_{\delta_m}^{\delta} u(u - u_0)\,dy \to 0, \ \frac{d}{dx}\int_{\delta_m}^{\delta} u\left(u^2 - u_0^2\right)dy \to 0 \quad (2.5)$$

as $\varepsilon \to 0$ and a fixed value of the blowing parameter m. The solution to the linearized system (2.4) indicates an absence of mixing in the main region of the wall jet when the thickness of the outer edge remains unchanged and equal to its thickness at the end of the initial segment of the jet

$$u_m = u_s(\varepsilon = \varepsilon_s), \ \delta - \delta_m = (\delta - \delta_m)_s. \quad (2.6)$$

The tendency to stabilize the growth of the outer region of the wall jet (2.6) is confirmed by experimental data [6] and, closer the velocities of the mixing flows, more accurately is thus fulfilled. For the wall boundary layer the momentum equation (2.3) along with (2.6) is transformed to the form

$$\frac{dr}{d\xi} = \frac{13}{15}r^{-2/13} \quad (2.7)$$

with initial conditions $r = 1$ at $\xi = 1$ and the solution $\bar{\delta}_m = \bar{\delta}_{m,s}\xi^{13/15}$, where $\bar{\delta}_{m,s} = \delta_{m,s}/s$ is the nondimensional wall boundary layer thickness at the end of the initial segment.

The interpolation formula for the efficiency of the gas curtain in the form of a power series $\eta = (1 + 1/\eta_{x\to\infty})^{-1}$ is used to eliminate the singularity at $x = 0$ which is present in the expression for the efficiency away from the location of blowing $\eta_{x\to\infty}$, and in the limit as $x \to \infty$ transforms to the expression (1.12). The inner and outer region thicknesses in (1.12) are determined from (2.6) and the solution to the equation (2.7). The use of the solutions to the system of linearized integral relations for the wall jet is possible from a comparison of the computed results for cooling efficiency with experimental data [2] (Fig. 2). Computations (carried out at $Re_s = 6000$ and $m = 1.5$) showed that the present theory gives good agreement with experiment in the range $1 < \frac{x}{s}\mathrm{Re}_s^{-0.25} < 20$ (curve 1). Computations based on the model of a developing turbulent wall boundary layer [2] give somewhat higher values of efficiency (curve 2). Unlike other methods to compute the efficiency of gas curtain [2] the present approach makes it possible to independently study the characteristics of the interaction of the wall layer and the outer mixing region of the curtain with the main flow in the initial and main segments and more fully take into consideration the influence of previous history of the flow on the temperature distribution in the flow past the adiabatic flat plate.

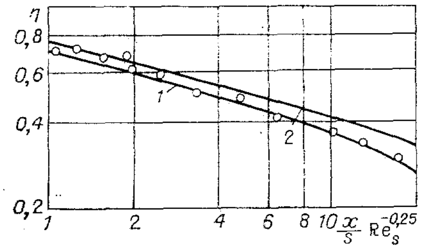

Fig. 2

3. Gas Curtain on Adiabatic Flat Plate with Pressure Gradient in the Main Flow ($R = \infty$, $u_0 = u_0(x)$). Limiting Mixing Laws. There is a fairly large amount of experimental data available in the literature today on the effect of positive and negative pressure gradients of the main flow on the efficiency of a gas curtain [2, 3] but there is no reliable computational method. Considering the same gas curtain model, viz., turbulent wall jet, the computational method for turbulent jets in flows with pressure gradient within the main segment is developed on the basis of a polynomial approximtion of the turbulent shear stress profiles [10] which requires a numerical solution of the resulting equations of motion. The method of integral relations is used here to find the solution using quadratures for the initial and main segments. There is a potential core in the initial segment (see Fig. 1), characterized by a constant total pressure which, according to the Bernoulli equation, leads to the condition of constant difference in the squares of velocities of the source and the main flow at any point in the initial segment [5]. Using velocity profiles (1.11), Eqs. (1.6)-(1.8) are integrated in a closed form and make it possible to obtain expressions for characteristic wall jet thickness in the initial segment

$$y_2 = \delta - \delta_1 = \left[\int_0^x \frac{b_3}{b_1} \exp\left(\int_0^x \frac{b_2}{b_1} dx\right) dx\right] \Bigg/ \exp\left[\int_0^x \frac{b_2}{b_1} dx\right]; \tag{3.1}$$

$$\bar{\delta}_m = 0.188\,\mathrm{Re}_s^{-2/15} \left[\int_0^x \bar{u}_s^{7/2}(x)\, dx\right]^{13/15} \bar{u}_s^{-19/6}, \tag{3.2}$$

where δ_1 is the ordinate of the outer edge of the potential core of the wall jet (see Fig. 1);

$$b_1 = (u_s - u_0)^3 a_3 + (u_s - u_0)^2 (2u_0 - u_s) a_2 - u_0 (u_s - u_0) a_1;$$

$$b_2 = \frac{d}{dx} [(u_s - u_0)^3 a_3 + (u_s - u_0)^2 (u_0 - 2u_s) a_2 + u_s (u_s - u_0)^2 a_1];$$

$$b_3 = -2k(u_s - u_0)^3 a_4; \quad \bar{u}_s(x) = u_s/u_{s,0}.$$

Here the reference velocity for the blowing velocity is the value at the nozzle section $u_{s,0}$. The length of the initial segment is found from the condition $\delta_1 = \delta_m = \delta_{m,s}$ that leads to the integral equation for the determination of x_s. It is necessary to mention that (3.1) is the universal expression for the free as well as for the wall jet, and there is no need to solve the integral equation of Volterra's second kind [5]. Numerical computations were carried out for a given variation of the main flow in the form of a linear relation $u_0(x) = u_{0,0}(1 + \gamma \bar{x})$, where $u_{0,0}$ is the velocity of the main flow at inlet, $\bar{x} = x/s$. Positive value of the parameters γ corresponds to a negative pressure gradient and the negative sign indicates positive gradient. An acceleration in the main flow is shown in Fig. 3 to lead to an increase in the initial segment whereas diffusion leads to a decrease. Curves 1-3 correspond to nondimensional velocity $\bar{u}_{0,0} = u_{0,0}/u_{s,0} = 1.25$; ; 1.5; 1.75. Similar qualitative behavior of the length of the initial segment x_s, depending on the streamwise pressure gradient, has been obtained for free jets [5]. Computations also showed an increase in wall boundary layer thickness at the end of the initial segment with an increase in the parameter γ (Fig. 4) which is also true for the thickness of the outer region (Fig. 5). The notations for curves in Figs. 4 and 5 are the same as in Fig. 3.

Using asymptotic approach to solve the integral relations (1.6) and (1.7) in the main segment, we get for the wall jet

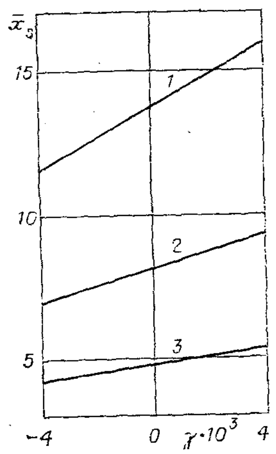

Fig. 3

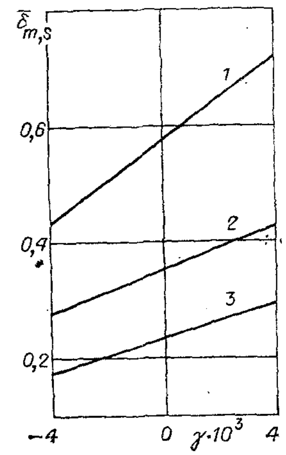

Fig. 4

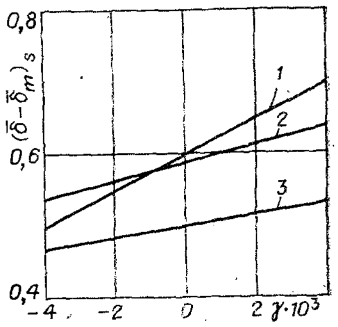

Fig. 5

$$\frac{d}{dx}\int_{\delta_m}^{\delta} u\left(u^2-u_0^2\right)dy\to 0,\quad \frac{d}{dx}\int_{\delta_m}^{\delta} u\,(u-u_0)\,dy+\frac{du_0}{dx}\int_{\delta_m}^{\delta}(u-u_0)\,dy\to 0 \tag{3.3}$$

as $\varepsilon \to 0$ and fixed m. The solution to the system (3.3) is expressed in the form of limiting mixing laws that determine the behavior of the mixing layer of the outer region of the turbulent wall jet:

$$\varepsilon=\varepsilon_s\frac{u_{0,0}}{u_0(x)},\quad \delta-\delta_m=(\delta-\delta_m)_s\frac{2a_1u_{0,0}+3a_2\varepsilon_s}{2a_1u_0+3a_2\varepsilon}. \tag{3.4}$$

Linearized momentum equation for the inner region similar to (2.7) is brought to the form

$$\frac{dr}{d\xi}=\frac{13}{15}\frac{1}{J}tr^{-2/13},\quad J=\int_0^1\bar{u}_s^{7/2}(\bar{x})\,d\bar{x}$$

with the initial condition r = 1 at ξ = 1 and the solution

$$r=\left[1+\left(\int_1^{\xi}t\,d\xi\right)\Big/J\right]^{13/15}. \tag{3.5}$$

Computations for the wall jet curtain using (3.4) and (3.5) and the rational interpolation formula used in p. 2 for the determination of efficient jet cooling showed that the presence of velocity gradient in the main flow does not practically affect the efficiency of the post-gradient cooling in the main segment.

4. Gas Curtain on Curvilinear Cylindrical Surface (R = const, $u_0 = u_0(x)$). Asymptotic Mixing Laws. The streamline curvature in the shear plane leads to changes in the structure of turbulence in the boundary layer: turbulence decreases near the convex surface (conservative effect of centrifugal forces) and increases in the concave region [7]. The corresponding change in momentum and heat is included either through the introduction of the Monin–Obukhov coefficient in the expression for the mixing length or the introduction of the additional relative Kutateladze – Leont'ev function [3, 10]. The consideration of streamwise surface curvature is especially significant in the computation of wall jets on curvilinear surfaces and the computation could be carried out on the basis of Karman momentum integral relations [8]. The analysis of terms on the right-hand side of Eqs. (1.6)-(1.8) is carried out using asymptotic mixing theory for the case u_0 = const. The strength of surface shear stresses and the integrals that take into account surface curvature in an explicit form becomes negligibly small:

$$\int_{\delta_m}^{\delta}\tau h^2\frac{\partial}{\partial y}(uh)\,dy\sim O(\varepsilon^3)\to 0,\quad \frac{1}{R}\int_{\delta_m}^{\delta}u\,dy\sim u_{0,w}\frac{1}{R}\frac{d}{dx}(\delta-\delta_m)\to 0,$$

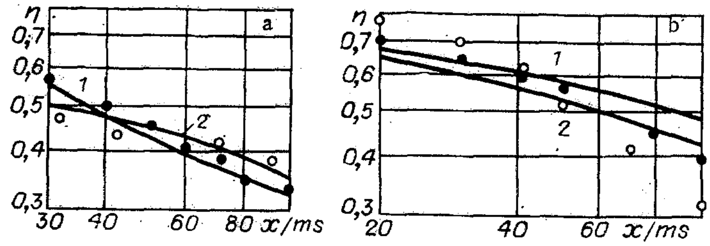

Fig. 6

$$\frac{1}{R}\int_{\delta_m}^{\delta} u\frac{\partial}{\partial x}\int_{\delta}^{y} u^2dy \sim \frac{1}{R}u_{0,w}^3\frac{d}{dx}(\delta-\delta_m)\to 0 \tag{4.1}$$

as $Re_x \to \infty$ and fixed m.

Results for the outer flow of the wall jet on a flat plate (2.6), having constant outer layer thickness under asymptotic conditions, are used in order to estimate the last two integrals in (4.1). The use of Eq. (2.6) for the estimation does not lead to large errors since it involves terms $O[(\delta/R)^2]$. When $\delta_m \ll \delta$ the expressions for the outer layer of the wall jet on curvilinear surface are written in the form (2.5) which makes it possible to use Eq. (2.6) to determine the maximum velocity and thickness of the outer layer. The determination of wall boundary-layer thickness, within the framework of asymptotic approximation, reduces to the integration of the equation of the type

$$\frac{d}{dx}\int_0^{\delta_m} u^2dy - u_m\frac{d}{dx}\int_0^{\delta_m} udy + u_m^2\frac{d\delta_m}{dx}\frac{\delta_m}{R} - u_{0,w}\frac{\delta_m}{R}\frac{d}{dx}\int_0^{\delta_m} udy + \tau_w/\rho = 0,$$

which, as in the case of (2.7) is reduced to the form

$$\left[1-\frac{91}{6}\frac{\delta_{m,s}}{R}\left(1-\frac{12}{13}\frac{u_{0,w}}{u_s}\right)r\right]\frac{dr}{d\xi} = \frac{13}{15}r^{-2/13}. \tag{4.2}$$

According to the method for an analytical solution, suggested in [8], the solution to (4.2) with initial conditions r = 1 at ξ = 1 will be the expression

$$r = \left\{\left[\xi - \frac{65}{8}\frac{\delta_{m,s}}{R}\left(1-\frac{12}{13}\bar{u}_0\right)\right]\Big/\left[1-\frac{65}{8}\frac{\delta_{m,s}}{R}\left(1-\frac{12}{13}\bar{u}_0\right)\xi^{13/15}\right]\right\}^{13/15}, \tag{4.3}$$

where $\bar{u}_0 = u_{0,w}/s$.

The influence of surface curvature is determined by the characteristics of the development of wall boundary layer on the curvilinear surface and the linear formulation of the problem does not permit the consideration of the effect of body forces in (4.1) on the turbulence characteristics of the outer layer of the wall jet. Expression (4.3) shows that the nature of the effect of surface curvature depends on the difference in circular brackets. If $u_{0,w} < u_m$, i.e., blowing rate is greater than the main flow velocity, the thickness of the wall boundary layer on the convex surface is greater than that on the concave surface. Here the cooling efficiency of the gas curtain, according to (1.12), should be less when m > 1 on the convex surface and more on the concave surface compared to a flat plate. Such a difference in the dynamic effect of centrifugal forces is explained by their stabilizing effects on concave and destabilizing effects on convex surfaces [9]. When the blowing coefficient is less than one ($u_{0,w} > u_m$), the surface curvature has an opposite effect: cooling efficiency of the gas curtain is higher on the convex surface and lower on the concave. Thus, in order to improve cooling efficiency with gas curtain on a curvilinear surface it is necessary to blow on the convex surface with a velocity less than that of the main stream; for concave surface the blowing coefficient should be greater than one.

The above-described technique is the basis of hydrodynamic control by efficient wall curtain on a curvilinear surface. Computations of wall-jet cooling efficiency carried out using

Eqs. (2.6) and (4.3) and interpolation formulas (p. 2 and 3) showed satisfactory agreement with experimental data [9]. In Figs. 6a, b dark points denote convex surface and light points denote concave surface. It is worth noting that computations at $m = 1.19$ (Fig. 6a) agree with experiments over a wider range of variation in the parameter $x/(ms)$ than when $m = 0.61$ (Fig. 6b), from 30 to 100 in the first case and from 30 to 60 in the second case. This is because when $m > 1$ the curtain retains the characteristics of a jet over a larger distance from the nozzle lip and, hence, is better described by theory based on jets. In the above ranges the maximum difference between computations and experiment does not exceed 10%. The effect of surface curvature is also clearly observed: cooling efficiency on convex surface (curve 1) is less than that on concave surface (curve 2) when $m > 1$ and more when $m < 1$. The expressions obtained here, which are valid near the nozzle, are an addition to Kutateladze–Leont'ev relation that agrees best with experimental data when $x/(ms) > 60$ [1]. Since the extent of the initial segment in the wall jets is less than that of free jets, it was assumed in computations that the parameters in the initial segment of wall jet along curvilinear surface were identical to the characteristics of the initial segment of wall jet on a plate plate.

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